Assignment 0

Reinforcement Learning

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Note: This is an ungraded assignment. Marks scored in this assignment will **not** be counted towards the final score.

1. There are n bins of which the kth contains $k-1$ blue balls and $n-k$ red balls. You pick a bin at random and remove two balls at random without replacement. Find the probability that:

   - the second ball is red;
   - the second ball is red, given that the first is red.

   (a) 1/3, 2/3
   (b) 1/2, 1/3
   (c) 1/2, 2/3
   (d) 1/3, 1/3

2. A medical company touts its new test for a certain genetic disorder. The false negative rate is small: if you have the disorder, the probability that the test returns a positive result is 0.999. The false positive rate is also small: if you do not have the disorder, the probability that the test returns a positive result is only 0.005. Assume that 2% of the population has the disorder. If a person chosen uniformly from the population is tested and the result comes back positive, what is the probability that the person has the disorder?

   (a) 0.803
   (b) 0.976
   (c) 0.02
   (d) 0.204

3. In an experiment, n coins are tossed, with each one showing up heads with probability p independently of the others. Each of the coins which shows up heads is then tossed again. What is the probability of observing 5 heads in the second round of tosses, if we toss 15 coins in the first round and $p = 0.4$?

   (Hint: First find the mass function of the number of heads observed in the second round.)

   (a) 0.372
   (b) 0.055
   (c) 0.0345

(d) 0.0488

4. An airline knows that 5 percent of the people making reservations on a certain flight will not show up. Consequently, their policy is to sell 52 tickets for a flight that can hold only 50 passengers. What is the probability that there will be a seat available for every passenger who shows up?

   (a) 0.5101
   (b) 0.81
   (c) 0.6308
   (d) 0.7405

5. Let X have mass function

$$f(x) = \begin{cases} \{x(x+1)\}^{-1} & \text{if x = 1, 2, ...,} \\ 0 & \text{otherwise,} \end{cases}$$

   and let $\alpha \in \mathbb{R}$. For what values of α is it the case that $\mathbb{E}(X^\alpha) < \infty$?

   (a) $\alpha < \frac{1}{2}$
   (b) $\alpha < 1$
   (c) $\alpha > 1$
   (d) $\alpha > \frac{3}{4}$

6. Is the following a distribution function?

$$F(x) = \begin{cases} e^{-1/x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

   If so, give the corresponding density function. If not, mention why it is not a distribution function.

   (a) No, not a monotonic function
   (b) Yes, $x^{-2}e^{-1/x}, x > 0$
   (c) No, not right continuous
   (d) Yes, $x^{-1}e^{-1/x}, x > 0$

7. Let $A^{m\times n}$ be a matrix of real numbers. The matrix AA^T has an eigenvector x with eigenvalue b. Then the eigenvector y of A^TA which has eigenvalue b is equal to

   (a) x^TA
   (b) A^Tx
   (c) x
   (d) Cannot be described in terms of x

8. Let $A^{n\times n}$ be a row stochastic matrix - in other words, all elements are non-negative and the sum of elements in every row is 1. Let b be an eigenvalue of A. Which of the following is true?

(a) $|b| > 1$

(b) $|b| <= 1$

(c) $|b| >= 1$

(d) $|b| < 1$

9. Let u be a $n \times 1$ vector, such that $u^T u = 1$. Let I be the $n \times n$ identity matrix. The $n \times n$ matrix A is given by $(I - kuu^T)$, where k is a real constant. u itself is an eigenvector of A, with eigenvalue -1. What is the value of k?

(a) -2

(b) -1

(c) 2

(d) 0

10. Which of the following are true for any $m \times n$ matrix A of real numbers

(a) The rowspace of A is the same as the columnspace of A^T

(b) The rowspace of A is the same as the rowspace of A^T

(c) The eigenvectors of AA^T are the same as the eigenvectors of $A^T A$

(d) The eigenvalues of AA^T are the same as the eigenvalues of $A^T A$

11. The Singular Value Decomposition (SVD) of a matrix R is given by USV^T. Consider an orthogonal matrix Q and $A = QR$. The SVD of A is given by $U_1 S_1 V_1^T$. Which of the following is/are true?

(a) $U = U_1$

(b) $S = S_1$

(c) $V = V_1$